About The Fourth Dimension

When you launch HyperCuber it draws a four-dimensional “hypercube,” an object analogous to a normal three-dimensional cube. A hypercube (and fourth-dimensional space in general) is best understood by analogy to the cube and three-space.

Note: It is sometimes said that time is the fourth dimension. Time can be regarded as a dimension, but it doesn’t have to be the fourth. Specifically, this program does not use time as its fourth dimension — the hypercube is in a space with four spatial dimensions. If time were also represented in the four-space this program uses, it would have to be the fifth dimension (or higher).

Back to analogies: a square is an object composed of four line segments of equal length; any two opposite segments are parallel to each other, and any two segments which touch are perpendicular. A cube is an object composed of six squares; any two opposite squares are parallel to each other, and any two squares which touch are perpendicular. Similarly, a hypercube is an object composed of eight cubes; any two opposite cubes are parallel to each other, and any two cubes which touch are perpendicular. Thus, the hypercube is precisely analogous to the square and the cube.

By now, if you have never encountered the fourth dimension, you should be spluttering something about cubes being parallel or perpendicular to each other. You see, having a fourth dimension tends to allow some truly bizarre things to happen. One such thing is that cubes can be either parallel or perpendicularly oriented with respect to each other (or be at any other angle, for that matter). It’s times like these that Flatland comes in handy.

Flatland is a “plane-world” with no concept of the third dimension. Creatures in Flatland are purely two-dimensional. They can move forward and backwards, or side-to-side, but not up and down. Furthermore, they have no idea what they’re missing— “up” and “down” are meaningless to them, since their entire world lacks the vertical dimension.

Now Flatlanders understand what it is for lines to be perpendicular— that’s when two lines are at a right angle in their plane-world. They also understand how two such lines can be parallel. But a Flatlander can’t understand how two squares can be parallel without lying on the same plane. Flatlanders, with their understanding of mathematics, know that non-coincident lines are parallel when they don’t intersect. They also understand, mathematically, that planes are parallel when they don’t intersect. But to the Flatlanders, the universe is all one plane, and every plane in the universe must be coincident with that one plane, so every square must lie within that one plane. They are unable to comprehend the existence of a plane which not only does not lie entirely in Flatland, but does not even touch it at all! Similarly they understand, mathematically, that if two planes are perpendicular, they must intersect in only one line, but they cannot imagine in what direction a plane must be rotated in order to achieve such an intersection with their world. Of course, we three-dimensional folks know that all you have to do is rotate one end of the plane “up” and it will cease to coincide with Flatland. But the Flatlanders don’t know anything about “up.”

We three-dimensional beings are just like Flatlanders when it comes to understanding the fourth dimension. Suppose I take two coincident cubes in three-space, say a meter to a side. That is, the cubes are in exactly the same plane, and intersect at every point. Then I say to you, “Please move one of the cubes one centimeter in any direction, making sure that when you’re done, the cubes do not intersect anywhere.” Perhaps you would then ship me off to have my head examined. But this is similar to asking a Flatlander to move one of two coincident squares so that they don’t touch afterwards. That’s easy for us— we just lift on of the squares by a centimeter. Then one is above the other, so they don’t touch anymore. Similarly, a four-dimensional being would look at the cube problem, and say, “No problem, just move it a centimeter that way,” while pointing, with some hideously distorted appendage, in a totally impossible direction.

So let’s talk about that “impossible direction” that we’re missing. Flatland is missing a direction, we know; it’s the “up” direction. It’s easy to construct the “up” direction (I’ll call it the z direction from now on, to be mathematically consistent); you take any perpendicular lines in Flatland, and find a line which is perpendicular to both. This is the z direction, and it extends perpendicularly out of Flatland. Note that any line in Flatland is perpendicular to this z direction.

It’s just as easy to describe how to find the direction we three-dimensional folks don’t know about (the w direction). You just take any three perpendicular lines in our space (say, the corner of a room), and find the line which is perpendicular to all three of those lines. This is the w direction, and a brief moment of thought will show you that it is, to us, a “totally impossible direction.” As in the case of Flatland, this w direction is perpendicular to any line in our space. It extends out of our space, perpendicularly.

So we finally return to our concept of “parallel cubes.” If you have two coincident cubes, and you want to make them parallel, just move one of them a little bit along the w direction and voilá! Parallel cubes. Similarly, two such cubes can be transformed into perpendicular cubes by rotating one of them out of our space in the direction of the w vector, until the w vector lies on one of the faces of the cube. And if you can visualize that, I'll buy you a beer.

The point that I've been trying to make, anyway, is that truly understanding the fourth dimension is not easy, and probably is not possible. The best I can do, personally, is to find three-dimensional analogies to everything I see. In the next section, I will explore some of these analogies.

Some Brain-Teasing Analogies

As I have said before, the fourth dimension can best be understood by analogy to the lower dimensions. Here are a few interesting facts about the fourth dimension, and their analogies in lower dimensions.

Fun 4D Fact
A line which passes through a cube will usually intersect in only one point (the exception occurring when the line lies in the same three-space as the cube).

3D Analogy:
A line which passes through a square will usually intersect only in one point (the exception occurring when the line lies in the same plane as the square).

Fun 4D Fact
Two non-parallel planes intersect in a point.

2D Analogy
Two non-parallel lines intersect in a point.

Fun 4D Fact
A four-dimensional "corner" is where four walls (3-spaces) meet in a single point, and all four walls are perpendicular to each other.

3D Analogy
A three-dimensional corner is where three walls (planes) meet in a single point, and all three walls are perpendicular to each other.

Fun 4D Fact
A hypercube is an object with eight cubes as sides, where each cube intersects and is perpendicular to six of the others, and is parallel to and does not intersect the seventh. The intersection between any cube and its perpendicular neighbors is a square.

3D Analogy
A cube is an object with six squares as sides, where each square intersects and is perpendicular to four of the others, and is parallel to and does not

intersect the fifth. The intersection between any square and its perpendicular neighbors is a line segment.

2D Analogy
A square is an object with four line segments of equal length as sides, where each line segment intersects and is perpendicular to two of the others, and is parallel to and does not intersect the third. The intersection between any segment and its perpendicular neighbors is a point.

and many more.... The fourth dimension is just like the third dimension; it just has one more dimension! This chapter could be extended to explain the fifth or the fiftieth dimension by analogy as well; this is left as an exercise for the reader (grin).